Maths Calculation Strategies taught at WTW

Why the workshop?

Strategies for teaching maths have changed since many of us were at school.

Previous approaches to teaching maths often did not support a secure, deep understanding of maths.

A recap: Key principles of a maths mastery approach

- The majority of pupils will move through the programmes of study at broadly the same pace
- *Depth* not acceleration
- Puts numbers first
- Using the correct maths vocabulary is an essential part of every lesson
- Concrete Pictorial Abstract
- Focuses on fluency, reasoning and problem solving (Calculate, Apply, Think)

Why not go straight to the standard formal methods?

1	0	4	3	2	8
+	6	1	7	3	1
1	6	6	0	5	9
		1			

	2	9	${}^{3}\cancel{4}$	${}^{1}3$	8	2
−	1	8	2	5	0	1
	1	1	1	8	8	1

TTh	Th	H	T	O
	2	7	3	9
×			2	8
2 ${}_{2}$	1 ${}_{5}$	9 ${}_{3}$	1 ${}_{7}$	2
5 ${}_{1}$	4	7 ${}_{1}$	8	0
7	6	6	9	2
		1		

$2,739 \times 28 = 76,692$

$372 \div 15 = 24 \text{ r}12$

			2	4	r	1	2
1	5	3	7	2			
	−	3	0	0			
			7	2			
	−		6	0			
			1	2			

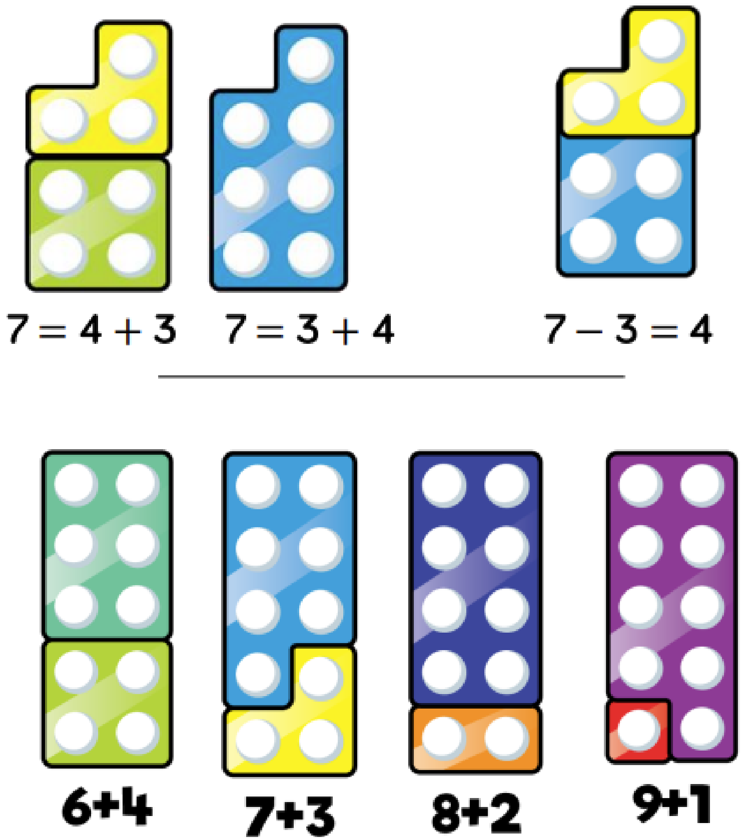
$1 \times 15 = 15$
$2 \times 15 = 30$
$3 \times 15 = 45$
$4 \times 15 = 60$
$5 \times 15 = 75$
$10 \times 15 = 150$

			2	4	$\frac{4}{5}$
1	5	3	7	2	
	−	3	0	0	
			7	2	
	−		6	0	
			1	2	

$372 \div 15 = 24\frac{4}{5}$

Strategies for Addition, Subtraction, Multiplication and Division

Number Shapes



Benefits

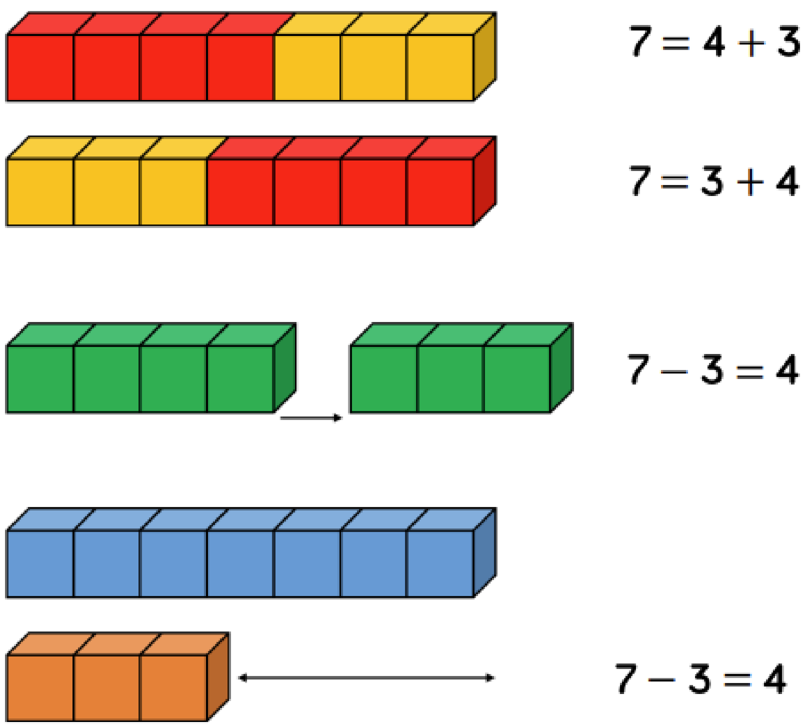
Number shapes can be useful to support children to subitise numbers as well as explore aggregation, partitioning and number bonds.

When adding numbers, children can see how the parts come together making a whole. As children use number shapes more often, they can start to subitise the total due to their familiarity with the shape of each number.

When subtracting numbers, children can start with the whole and then place one of the parts on top of the whole to see what part is missing. Again, children will start to be able to subitise the part that is missing due to their familiarity with the shapes.

Children can also work systematically to find number bonds. As they increase one number by 1, they can see that the other number decreases by 1 to find all the possible number bonds for a number.

Cubes



Benefits

Cubes can be useful to support children with the addition and subtraction of one-digit numbers.

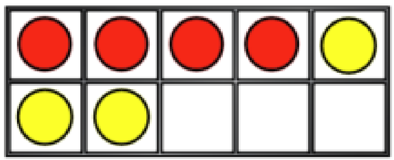
When adding numbers, children can see how the parts come together to make a whole. Children could use two different colours of cubes to represent the numbers before putting them together to create the whole.

When subtracting numbers, children can start with the whole and then remove the number of cubes that they are subtracting in order to find the answer. This model of subtraction is reduction, or take away.

Cubes can also be useful to look at subtraction as difference. Here, both numbers are made and then lined up to find the difference between the numbers.

Cubes are useful when working with smaller numbers but are less efficient with larger numbers as they are difficult to subitise and children may miscount them.

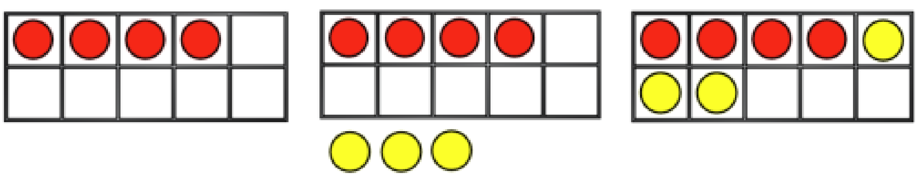
Ten Frames (within 10)



$4 + 3 = 7$
$3 + 4 = 7$
$7 - 3 = 4$
$7 - 4 = 3$

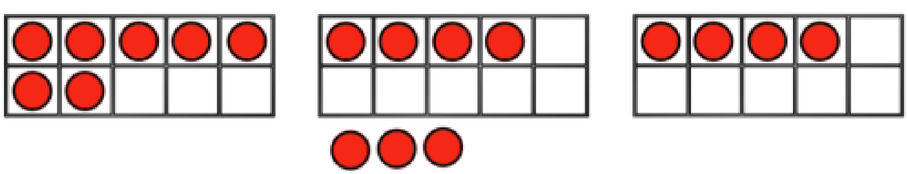
4 is a part.
3 is a part.
7 is the whole.

First　　Then　　Now



$4 + 3 = 7$

First　　Then　　Now



$7 - 3 = 4$

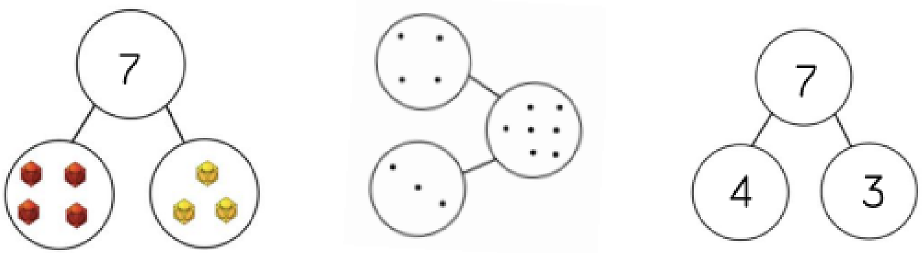
Benefits

When adding and subtracting within 10, the ten frame can support children to understand the different structures of addition and subtraction.

Using the language of parts and wholes represented by objects on the ten frame introduces children to aggregation and partitioning.
Aggregation is a form of addition where parts are combined together to make a whole. Partitioning is a form of subtraction where the whole is split into parts. Using these structures, the ten frame can enable children to find all the number bonds for a number.

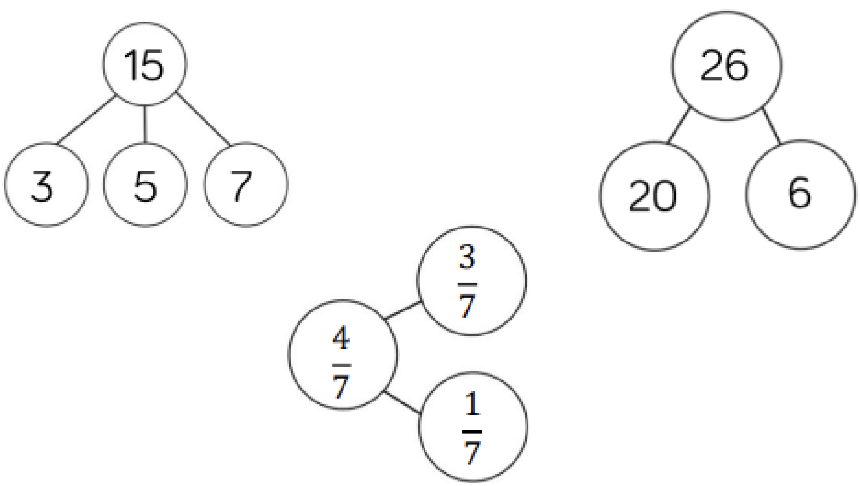
Children can also use ten frames to look at augmentation (increasing a number) and take-away (decreasing a number). This can be introduced through a first, then, now structure which shows the change in the number in the 'then' stage. This can be put into a story structure to help children understand the change e.g. First, there were 7 cars. Then, 3 cars left. Now, there are 4 cars.

Part-Whole Model



$7 = 4 + 3$
$7 = 3 + 4$

$7 - 3 = 4$
$7 - 4 = 3$



Benefits

This part-whole model supports children in their understanding of aggregation and partitioning. Due to its shape, it can be referred to as a cherry part-whole model.

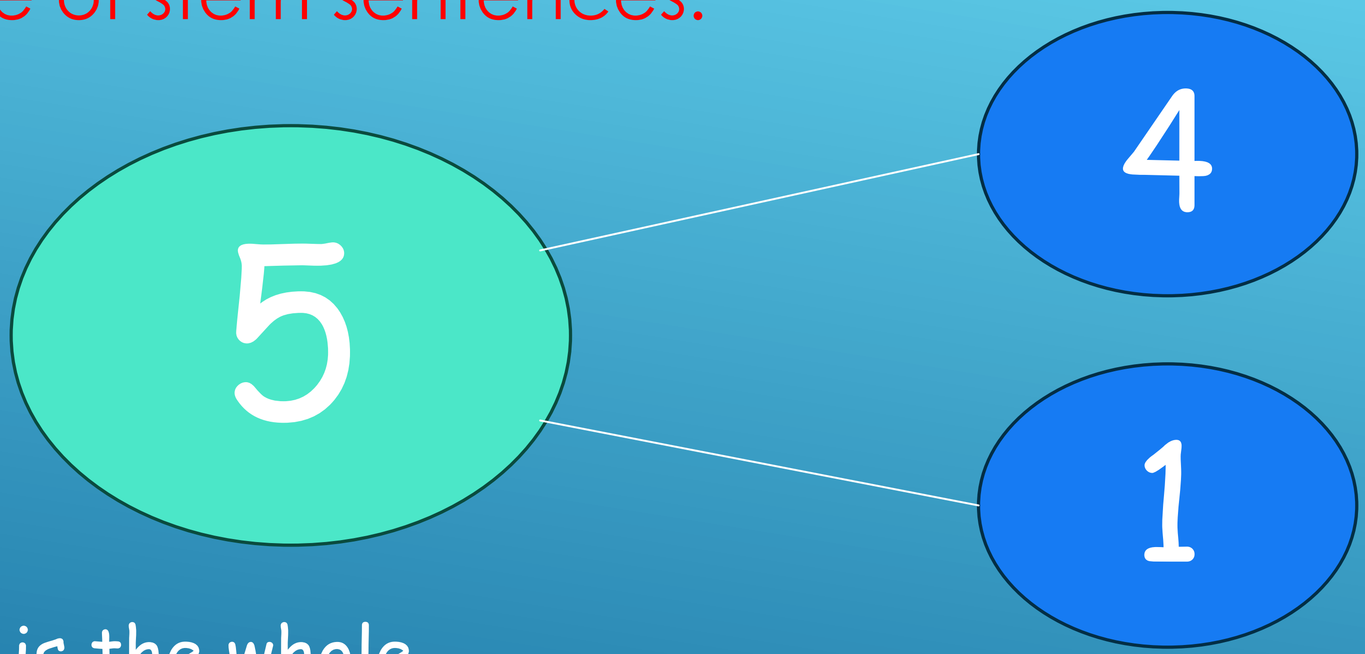
When the parts are complete and the whole is empty, children use aggregation to add the parts together to find the total.

When the whole is complete and at least one of the parts is empty, children use partitioning (a form of subtraction) to find the missing part.

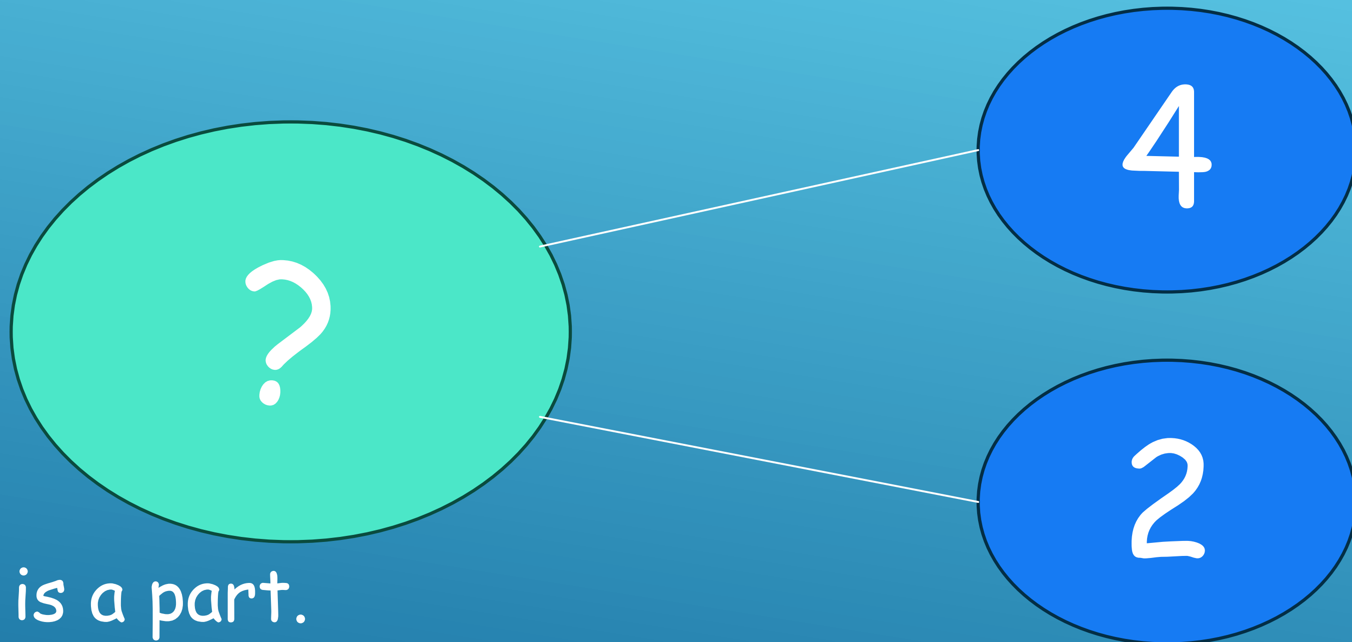
Part-whole models can be used to partition a number into two or more parts, or to help children to partition a number into tens and ones or other place value columns.

In KS2, children can apply their understanding of the part-whole model to add and subtract fractions, decimals and percentages.

Use of stem sentences.



5 is the whole.
4 is a part.
1 is a part.



4 is a part.
2 is a part.
6 is the whole

Subtraction within a context

Before

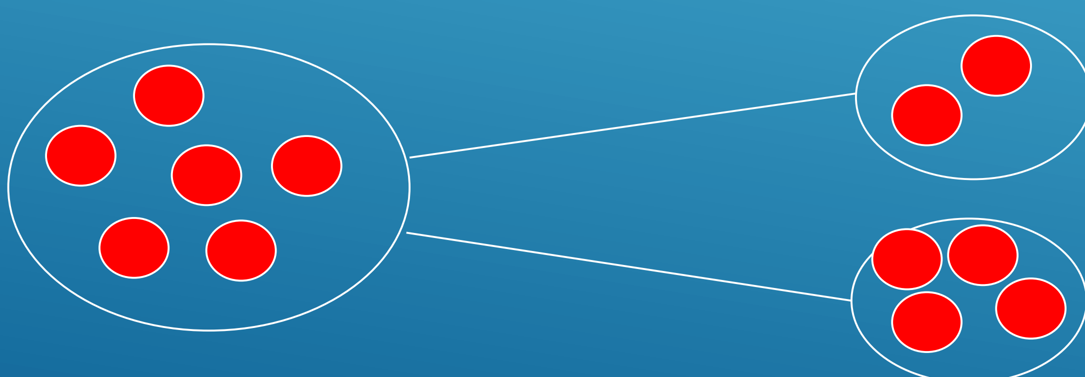
Next

Now

What does the 6 represent?

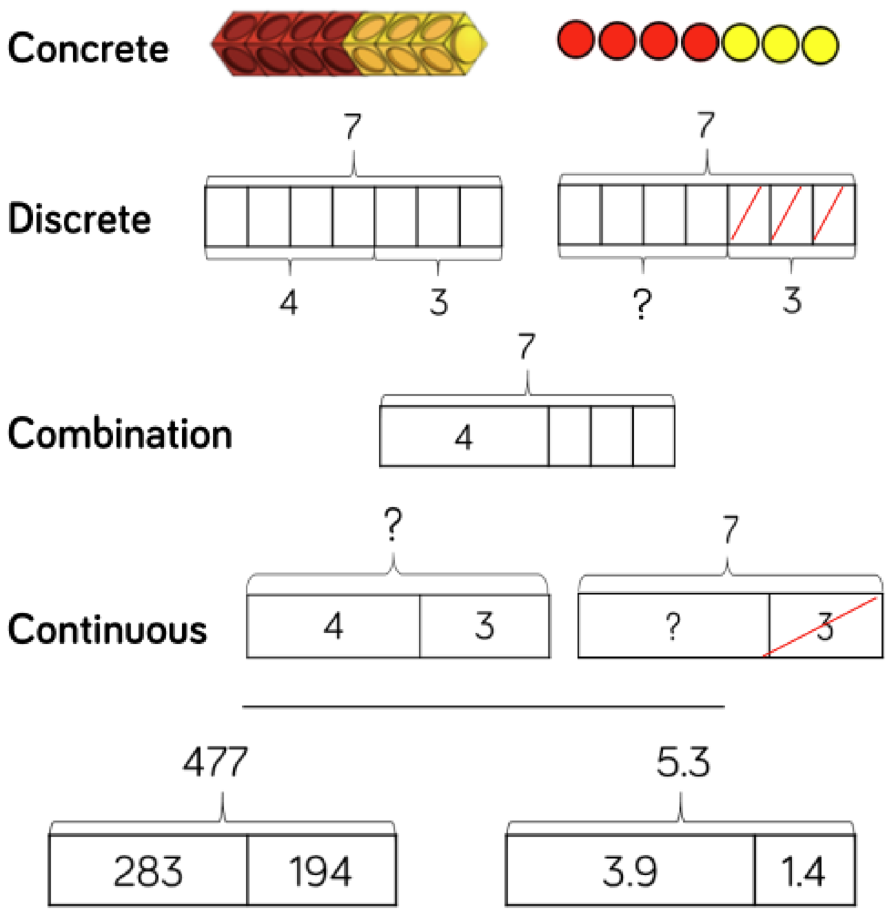
What does the 2 represent?

What does the 4 represent?



6 - 2 = 4

Bar Model (single)



Benefits

The single bar model is another type of a part-whole model that can support children in representing calculations to help them unpick the structure.

Cubes and counters can be used in a line as a concrete representation of the bar model.

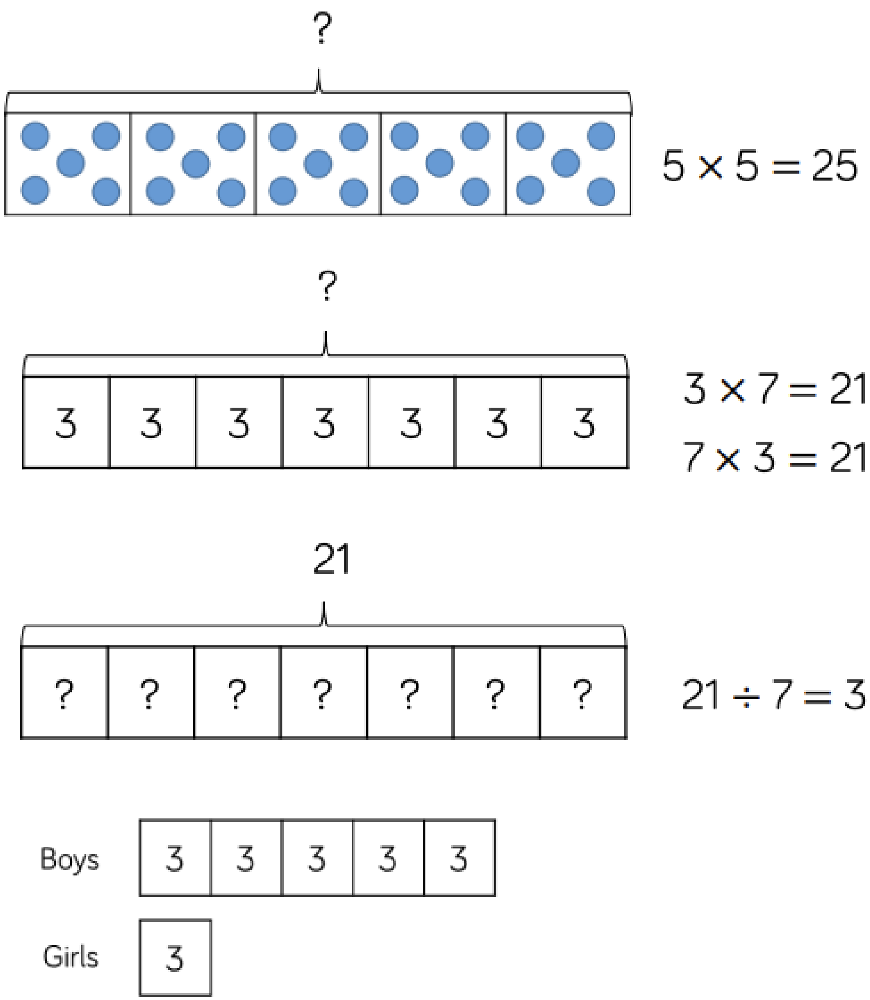
Discrete bar models are a good starting point with smaller numbers. Each box represents one whole.

The combination bar model can support children to calculate by counting on from the larger number. It is a good stepping stone towards the continuous bar model.

Continuous bar models are useful for a range of values. Each rectangle represents a number. The question mark indicates the value to be found.

In KS2, children can use bar models to represent larger numbers, decimals and fractions.

Bar Model



Benefits

Children can use the single bar model to represent multiplication as repeated addition. They could use counters, cubes or dots within the bar model to support calculation before moving on to placing digits into the bar model to represent the multiplication.

Division can be represented by showing the total of the bar model and then dividing the bar model into equal groups.

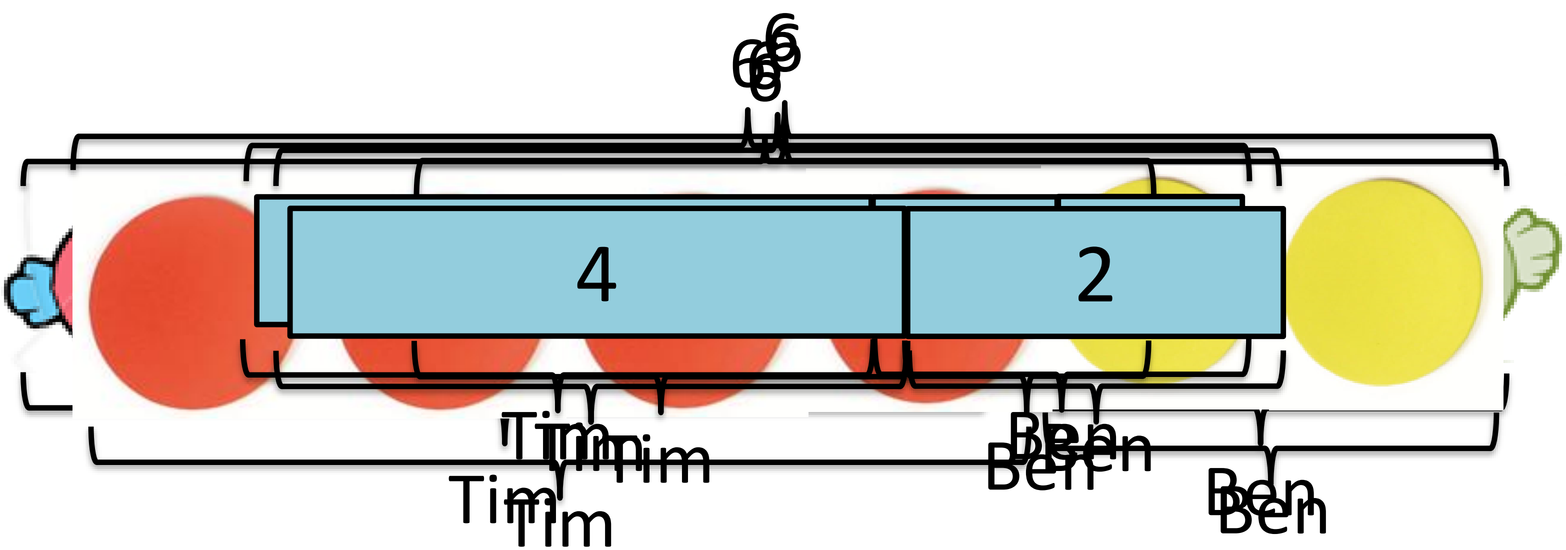
It is important when solving word problems that the bar model represents the problem.

Sometimes, children may look at scaling problems. In this case, more than one bar model is useful to represent this type of problem, e.g. There are 3 girls in a group. There are 5 times more boys than girls. How many boys are there?
The multiple bar model provides an opportunity to compare the groups.

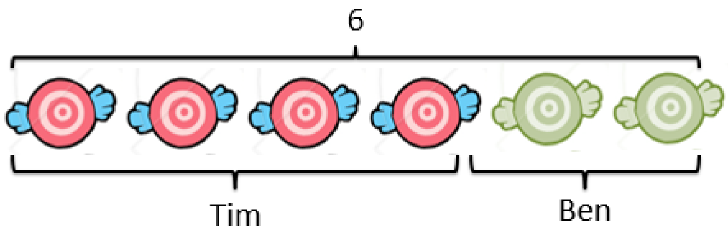
KS1 Bar Modelling

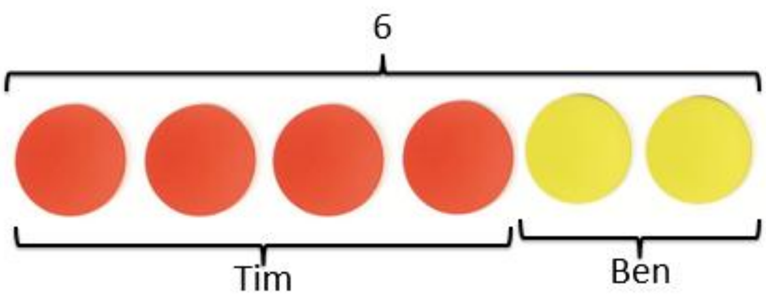
Tim has 4 sweets and Ben has 2 sweets.

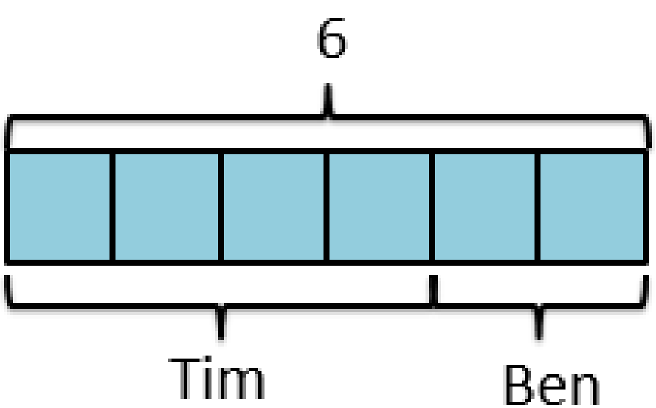
How many sweets do they have altogether?

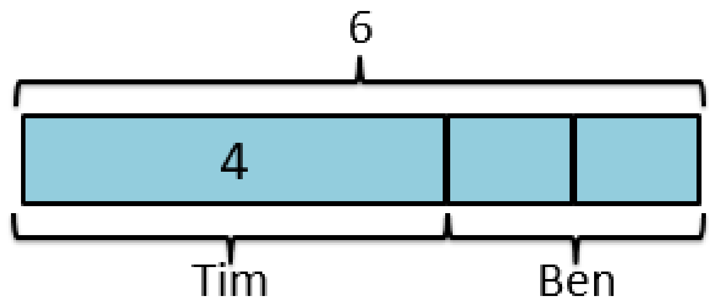


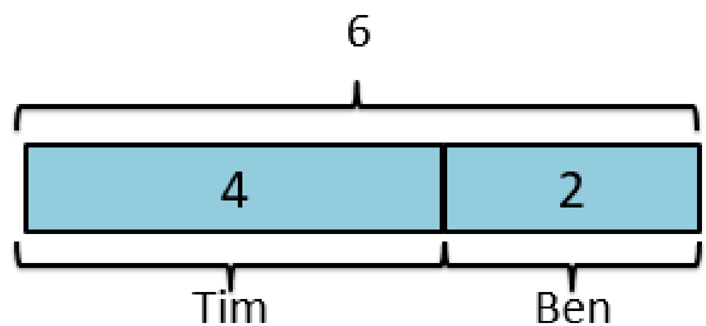
Small steps











$$4 + 2 = 6$$

KS2 barmodelling

MathsHUBS White Rose

$$\frac{3}{5} \text{ of } 20 = ?$$

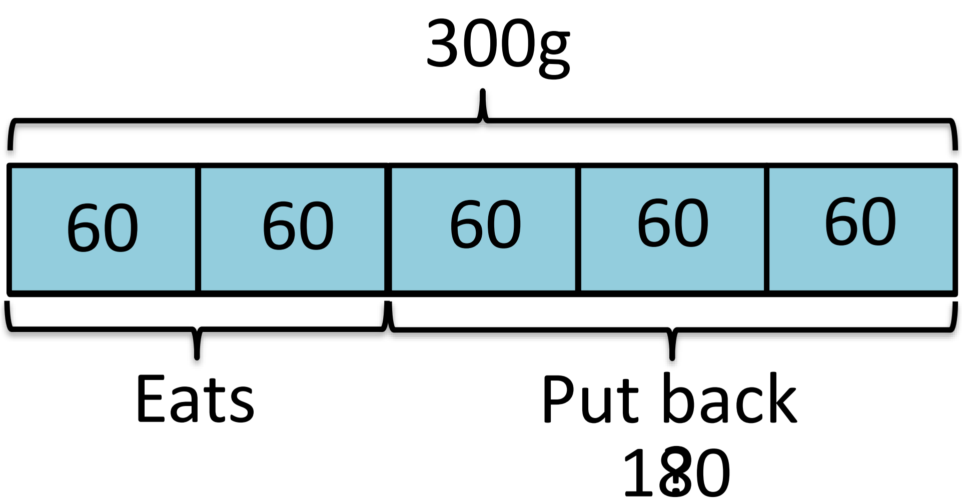
KS2 Bar Modelling

MathsHUBS White Rose

Solve… Matthew has a 300g block of cheese. He eats $\frac{2}{5}$ of the cheese and puts the rest back in the fridge.

How much cheese did Matthew put back in the fridge?

Model



Calculations

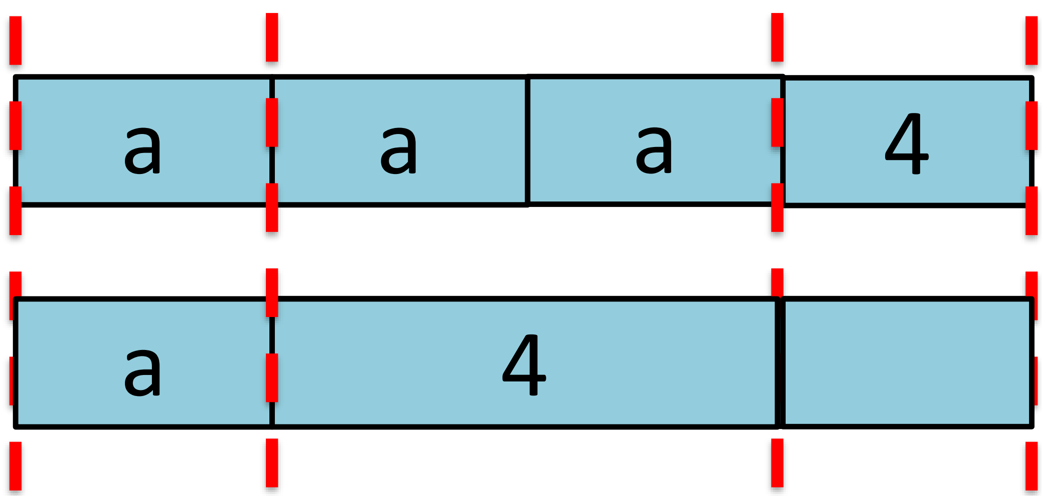
$300 \div 5 = 60$

$3 \times 60 = 180$

KS3 Bar Modelling

$3a + 4 = a + 8$

Model



Calculations

$$3a + 4 = a + 8$$

-a -a

$$2a + 4 = 8$$

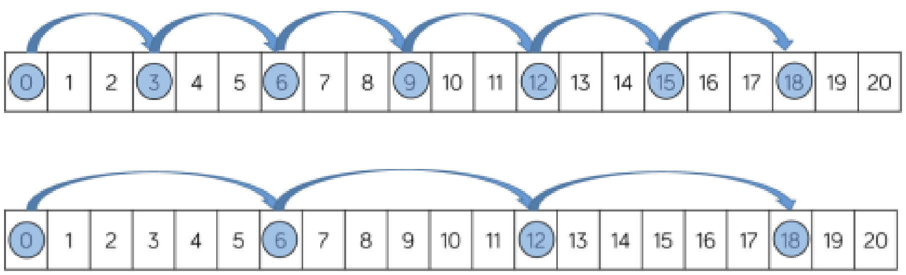
-4 -4

$$2a = 4$$

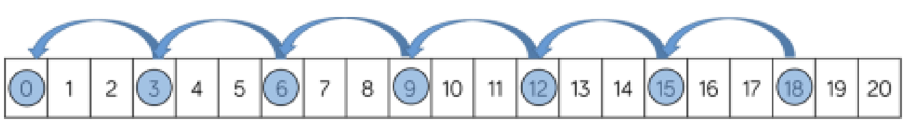
÷2 ÷2

$$a = 2$$

Number Tracks



$$6 \times 3 = 18$$
$$3 \times 6 = 18$$



$$18 \div 3 = 6$$

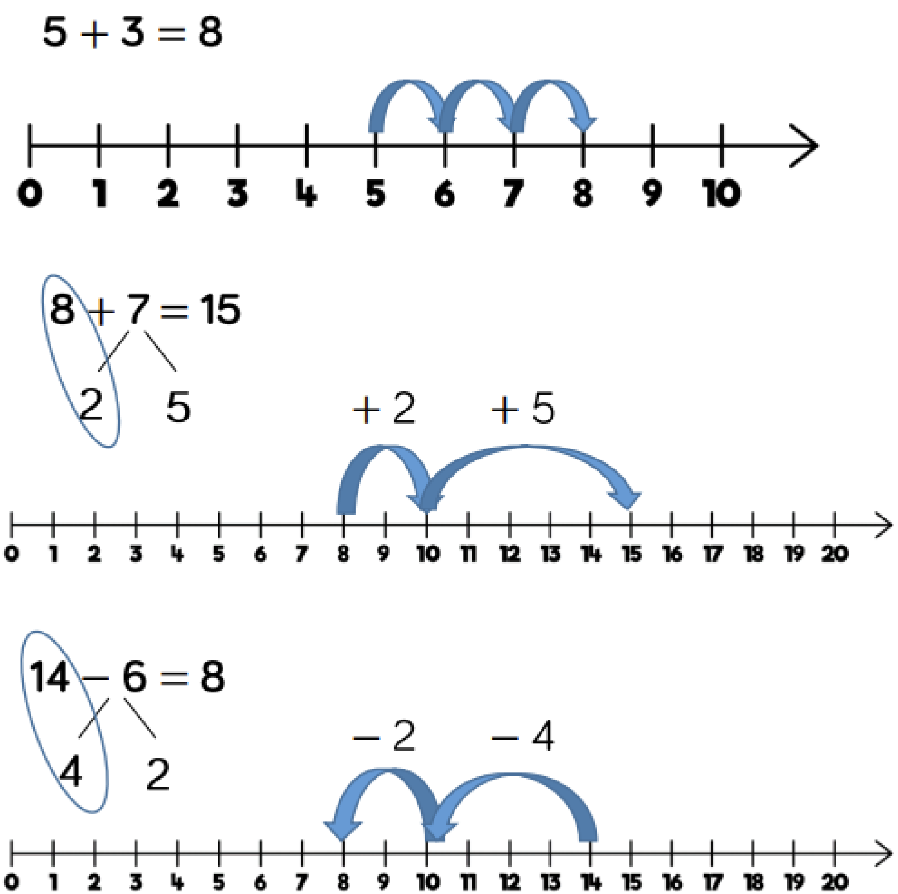
Benefits

Number tracks are useful to support children to count in multiples, forwards and backwards. Moving counters or cubes along the number track can support children to keep track of their counting. Translucent counters help children to see the number they have landed on whilst counting.

When multiplying, children place their counter on 0 to start and then count on to find the product of the numbers.
When dividing, children place their counter on the number they are dividing and the count back in jumps of the number they are dividing by until they reach 0. Children record how many jumps they have made to find the answer to the division.

Number tracks can be useful with smaller multiples but when reaching larger numbers they can become less efficient.

Number Lines (labelled)



Benefits

Labelled number lines support children in their understanding of addition and subtraction as augmentation and reduction.

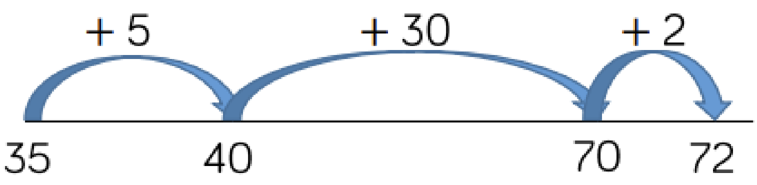
Children can start by counting on or back in ones, up or down the number line. This skill links directly to the use of the number track.

Progressing further, children can add numbers by jumping to the nearest 10 and then jumping to the total. This links to the making 10 method which can also be supported by ten frames. The smaller number is partitioned to support children to make a number bond to 10 and to then add on the remaining part.

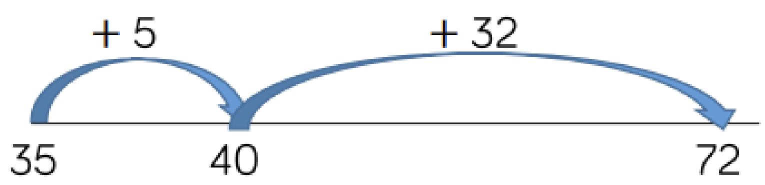
Children can subtract numbers by firstly jumping to the nearest 10. Again, this can be supported by ten frames so children can see how they partition the smaller number into the two separate jumps.

Number Lines (blank)

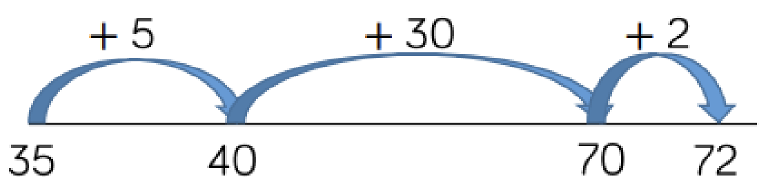
35 + 37 = 72



35 + 37 = 72



72 – 35 = 37



Benefits

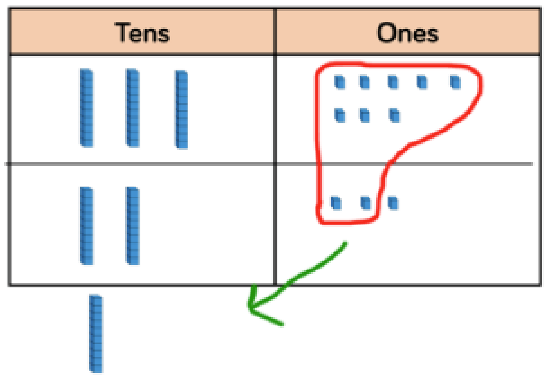
Blank number lines provide children with a structure to add and subtract numbers in smaller parts.

Developing from labelled number lines, children can add by jumping to the nearest 10 and then adding the rest of the number either as a whole or by adding the tens and ones separately.

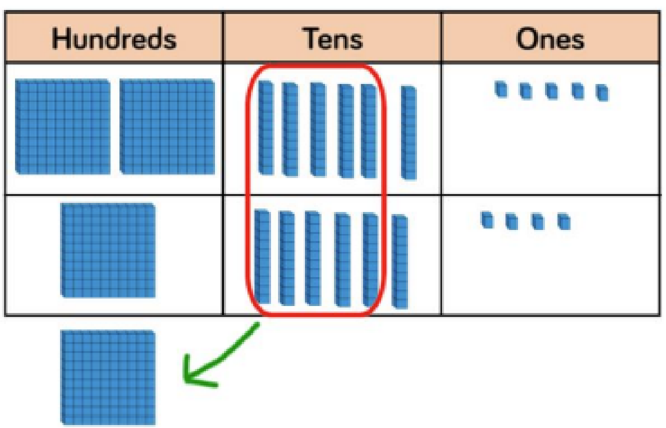
Children may also count back on a number line to subtract, again by jumping to the nearest 10 and then subtracting the rest of the number.

Blank number lines can also be used effectively to help children subtract by finding the difference between numbers. This can be done by starting with the smaller number and then counting on to the larger number. They then add up the parts they have counted on to find the difference between the numbers.

Base 10/Dienes (addition)



$$\begin{array}{r} 38 \\ +\ 23 \\ \hline 61 \\ \hline {\scriptstyle 1} \end{array}$$



$$\begin{array}{r} 265 \\ +\ 164 \\ \hline 429 \\ \hline {\scriptstyle 1}\ \ \end{array}$$

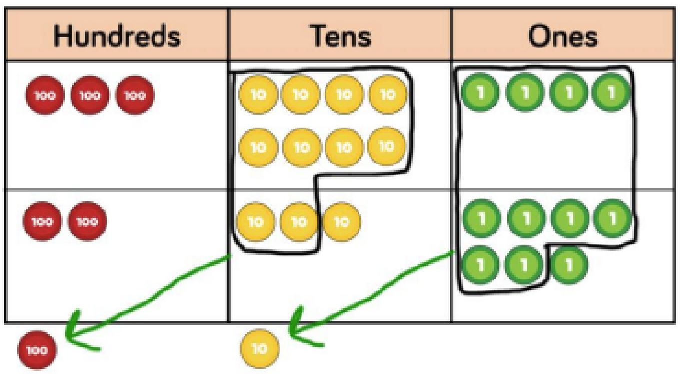
Benefits

Using Base 10 or Dienes is an effective way to support children's understanding of column addition. It is important that children write out their calculations alongside using or drawing Base 10 so they can see the clear links between the written method and the model.

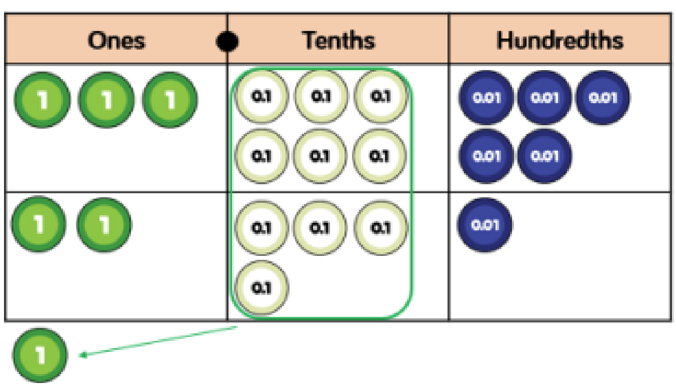
Children should first add without an exchange before moving on to addition with exchange.. The representation becomes less efficient with larger numbers due to the size of Base 10. In this case, place value counters may be the better model to use.

When adding, always start with the smallest place value column. Here are some questions to support children.
How many ones are there altogether?
Can we make an exchange? (Yes or No)
How many do we exchange? (10 ones for 1 ten, show exchanged 10 in tens column by writing 1 in column)
How many ones do we have left? (Write in ones column)
Repeat for each column.

Place Value Counters (addition)



$$
\begin{array}{r}
384 \\
+\,237 \\
\hline
621 \\
\hline
{\scriptstyle 1\ 1\ }
\end{array}
$$



$$
\begin{array}{r}
3.65 \\
+\,2.41 \\
\hline
6.06 \\
\hline
{\scriptstyle 1\ \ \ \ \ }
\end{array}
$$

Benefits

Using place value counters is an effective way to support children's understanding of column addition. It is important that children write out their calculations alongside using or drawing counters so they can see the clear links between the written method and the model.

Children should first add without an exchange before moving on to addition with exchange. Different place value counters can be used to represent larger numbers or decimals. If you don't have place value counters, use normal counters on a place value grid to enable children to experience the exchange between columns.

When adding money, children can also use coins to support their understanding. It is important that children consider how the coins link to the written calculation especially when adding decimal amounts.

Let's look at one operation in more depth …

multiplication

Multiplication in Reception

In Reception, children will learn:

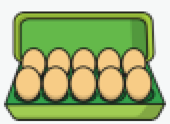
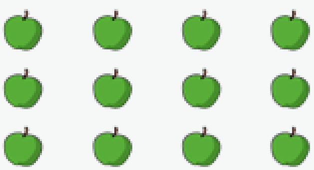
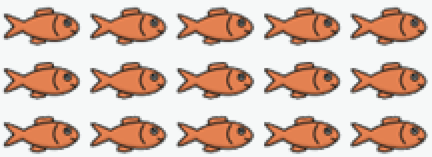
- Doubling to 10
- Make equal groups

Multiplication in Year 1

- Add equal groups

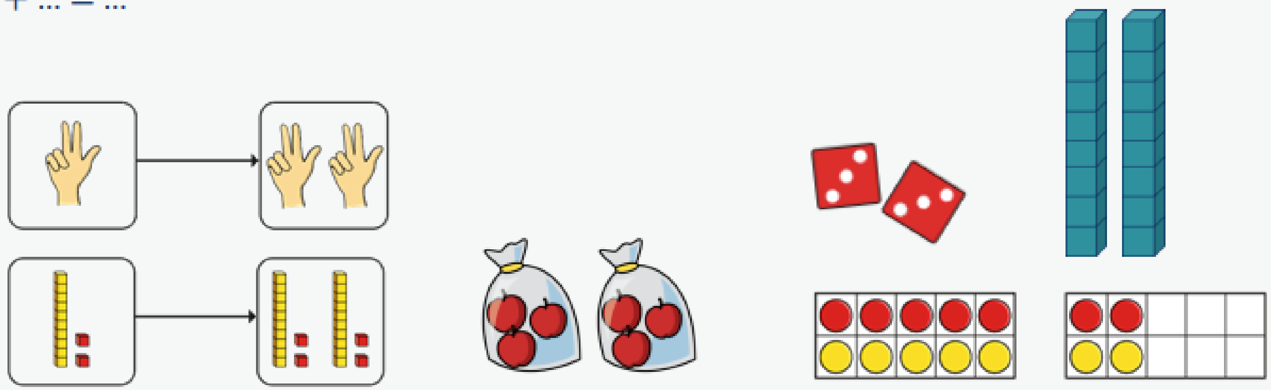
Add equal groups (repeated addition) Children should be able to write a repeated addition to represent equal groups and to draw pictures or use objects to represent a repeated addition.	There are … groups of … There are … altogether.  10 + 10 + 10 = 30  5 + 5 + 5 + 5 = 20

- Make arrays

Make arrays Children use their knowledge of adding equal groups to arrange objects in columns and rows.	There are … rows of … There are … altogether. There are … columns of … There are … altogether.   

Multiplication in Year 1

- Make doubles

Make doubles Children understand that doubles are two equal groups. Children may begin to explore doubles beyond 20 using base 10	Double … is … … + … = … 

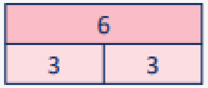
Multiplication in Year 2

- Link repeated addition and multiplication

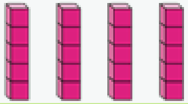
Link repeated addition and multiplication

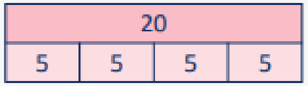
Encourage children to make the link between repeated addition and multiplication.

There are … equal groups with … in each group.
There are … altogether.



$3 + 3 = 6$
$2 \times 3 = 6$





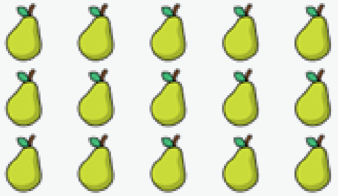
$5 + 5 + 5 + 5 = 20$
$4 \times 5 = 20$

- Use arrays

Use arrays

Encourage children to see that multiplication is commutative.

There are … rows with … in each row.
There are … columns with … in each column.



3 lots of 5 = 15
$5 + 5 + 5 = 15$

5 lots of 3 = 15
$3 + 3 + 3 + 3 + 3 = 15$

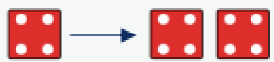
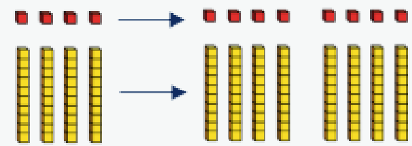
I can see … × … and … ×…

$3 \times 5 = 15$
$5 \times 3 = 15$

$3 \times 5 = 5 \times 3$

Multiplication in Year 2

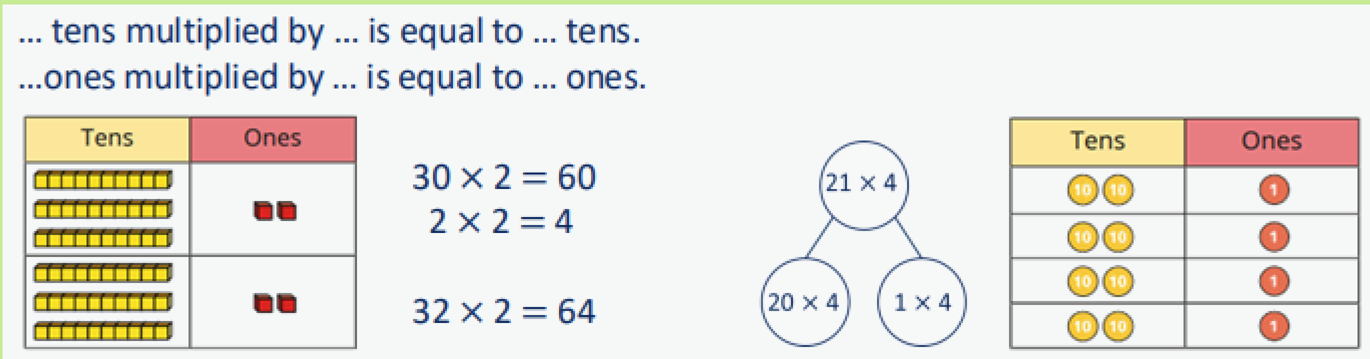
- Double

Double	Double … is …	Double … is … so double … is …
Encourage children to make links with related facts.	 Double 4 = 4 + 4 Double 4 is 8	 Double 4 is 8 Double 40 is 80

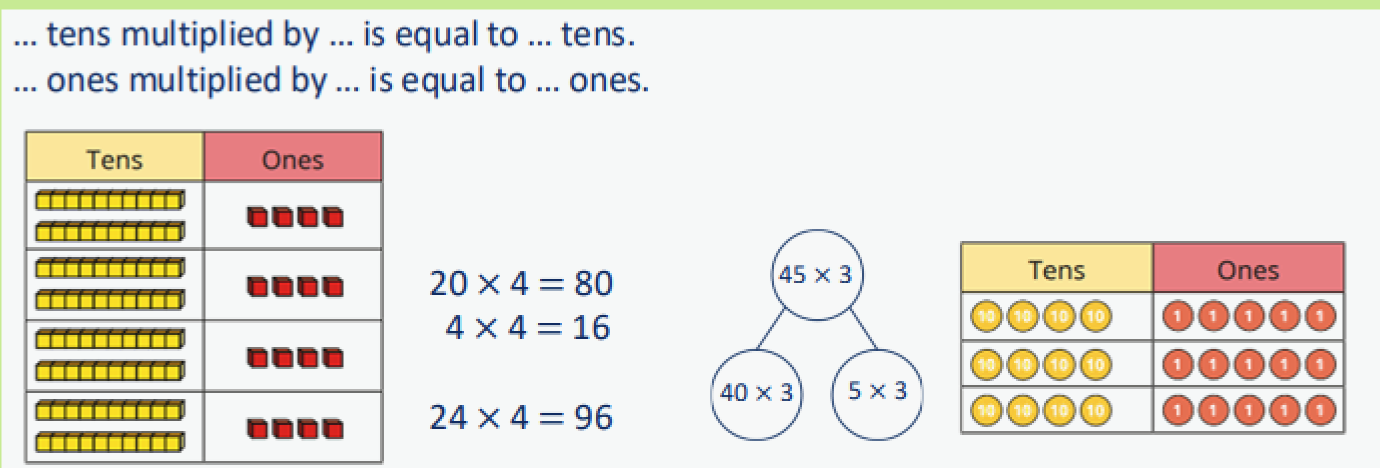
- Learn 2, 5 and 10 times tables

Multiplication in Year 3

- Multiply a 2-digit number by a 1-digit number (no exchange)



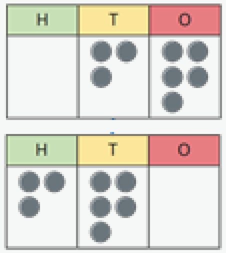
- Multiply a 2-digit number by a 1-digit number (with exchange)



Multiplication in Year 4

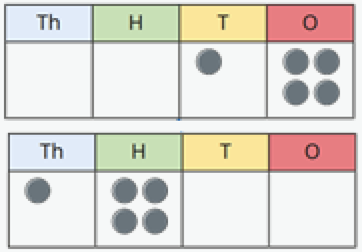
- ## Multiply by 10 and 100

When I multiply by 10, the digits move … place value column to the left.
… is 10 times the size of …



$35 \times 10 = 350$

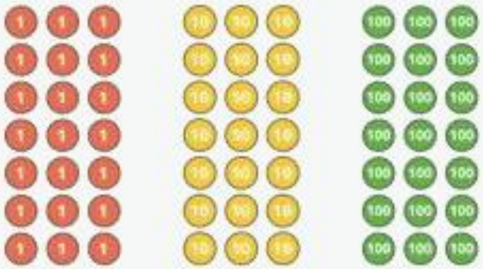
When I multiply by 100, the digits move … place value columns to the left.
… is 100 times the size of …



$14 \times 100 = 1{,}400$

- ## Related Facts

… × … ones is equal to … ones
so … × … tens is equal to … tens
and … × … hundreds is equal to … hundreds.

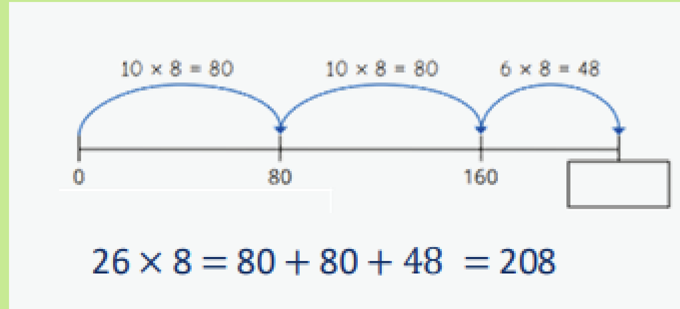


$3 \times 7 = 21$
$3 \times 70 = 210$
$3 \times 700 = 2{,}100$

$7 \times 3 = 21$
$7 \times 30 = 210$
$7 \times 300 = 2{,}100$

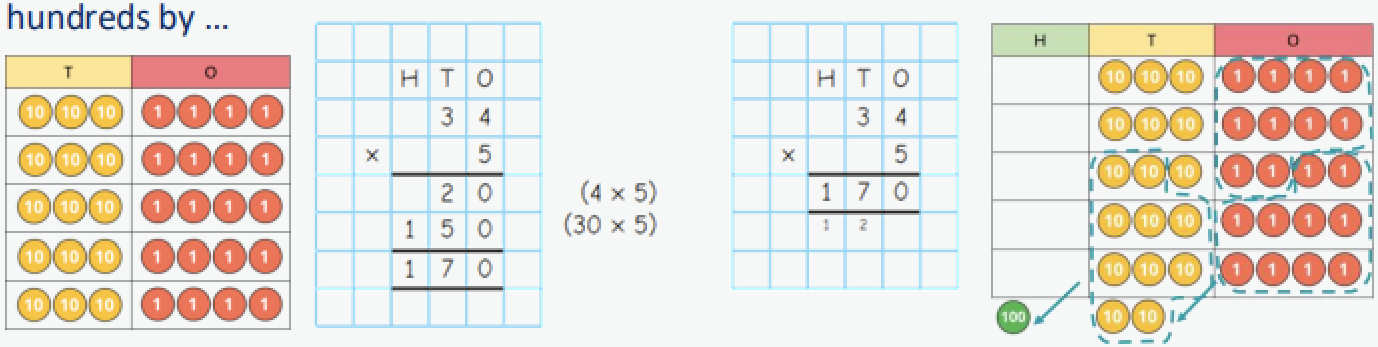
Multiplication in Year 4

- Mental strategies, eg:



- Multiply a 2 or 3-digit number by a 1-digit number (this is the first time the short method is introduced)

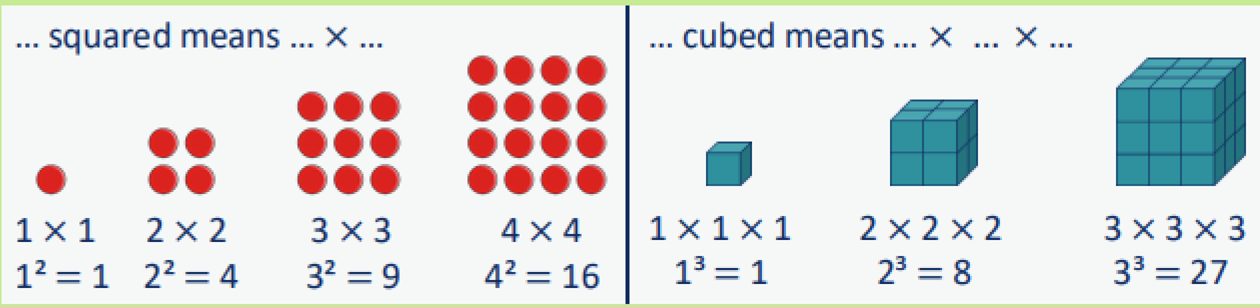
To multiply a 2-digit number by … , I multiply the ones by … and the tens by …
To multiply a 3-digit number by … , I multiply the ones by … , the tens by … and the hundreds by …



- Learn all times tables to 12 x 12

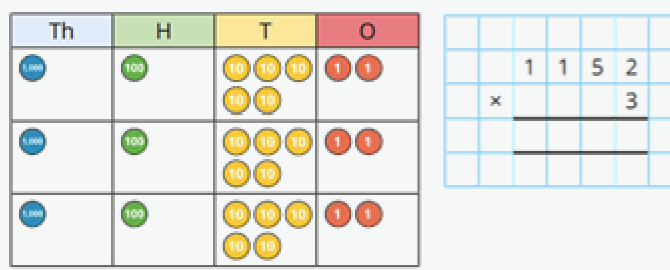
Multiplication in Year 5

- Square and cube numbers



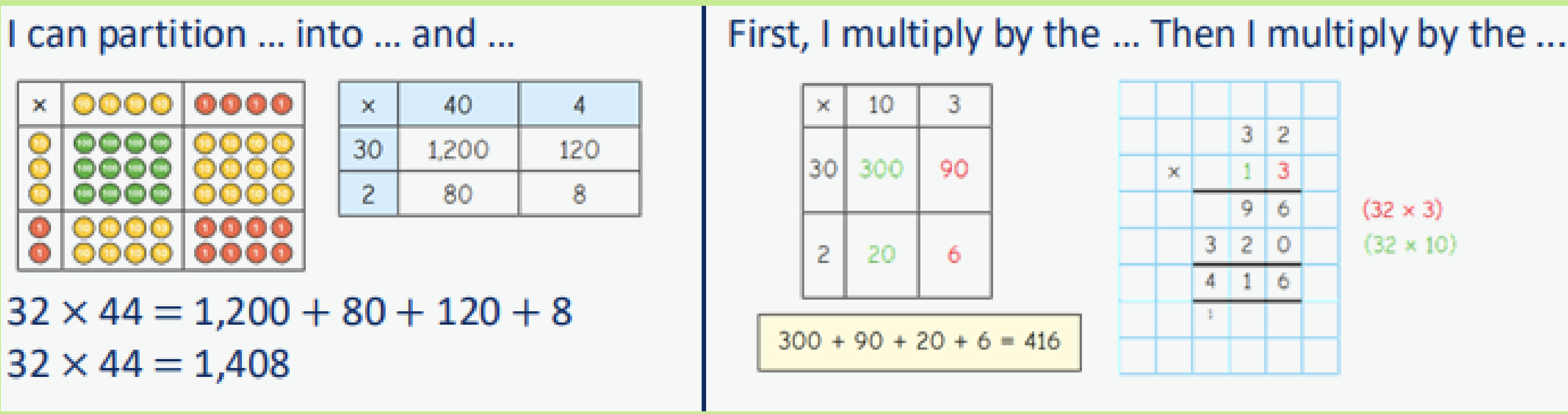
- Multiply numbers up to 4 digits by a 1-digit number

To multiply a 4-digit number by … , I multiply the ones by … , the tens by … , the hundreds by … and the thousands by …



Multiplication in Year 5

- Multiply numbers up to 4 digits by a 2-digit number (numbers are first partitioned using a ‘grid’ model, then long multiplication is introduced for the first time)



I can partition ... into ... and ...

×	40	4
30	1,200	120
2	80	8

$32 \times 44 = 1{,}200 + 80 + 120 + 8$

$32 \times 44 = 1{,}408$

First, I multiply by the ... Then I multiply by the ...

×	10	3
30	300	90
2	20	6

300 + 90 + 20 + 6 = 416

		3	2	
×		1	3	
		9	6	(32 × 3)
	3	2	0	(32 × 10)
	4	1	6	
	1			

- Multiply by 10, 100 and 1000

Multiplication in Year 6

- Multiply numbers up to 4 digits by a 2-digit number

To multiply by a 2-digit number, first multiply by the ones, then multiply by the tens and then find the total.

		1	2	0	7		
	×			3	6		
+		7	2	4	2		(1,207 × 6)
		1		4			
	3	6	2	1	0		(1,207 × 30)
			2				
	4	3	4	5	2		
	1						

- Multiply by 10, 100 and 1000

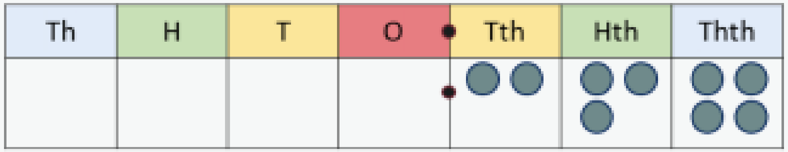
To multiply by 10/100/1,000, I move all the digits … places to the left.

… is 10/100/1,000 times the size of …

234 × 10 = 2,340

234 × 100 = 23,400

234 × 1,000 = 234,000



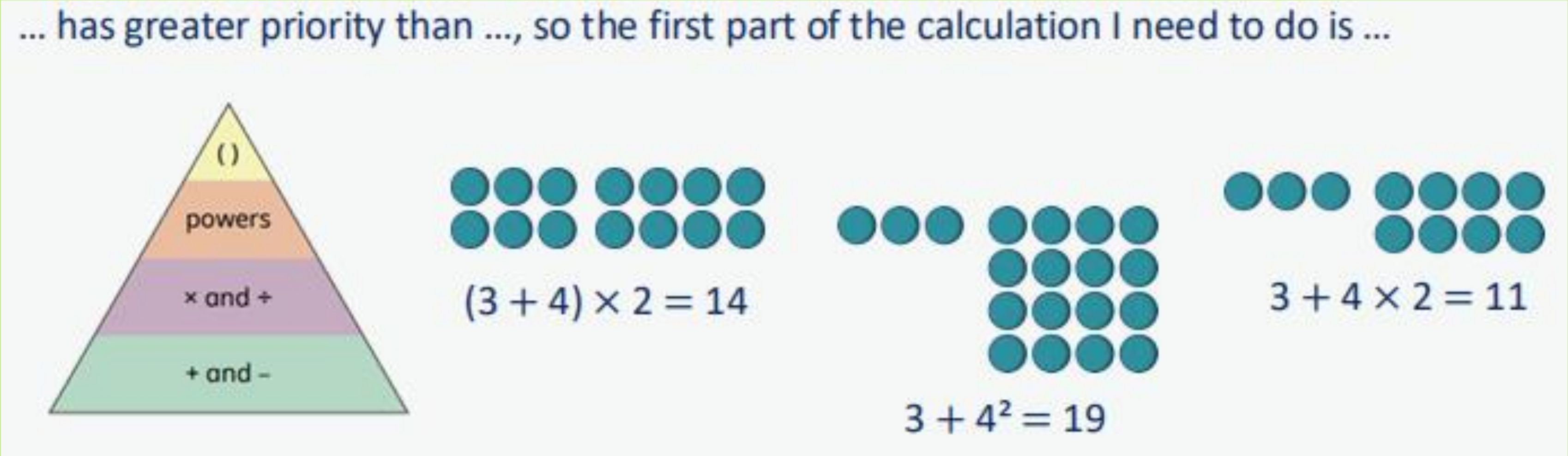
0.234 × 10 = 2.34

0.234 × 100 = 23.4

0.234 × 1,000 = 234

Multiplication in Year 6

- Order of operations



The children will learn this as BIDMAS